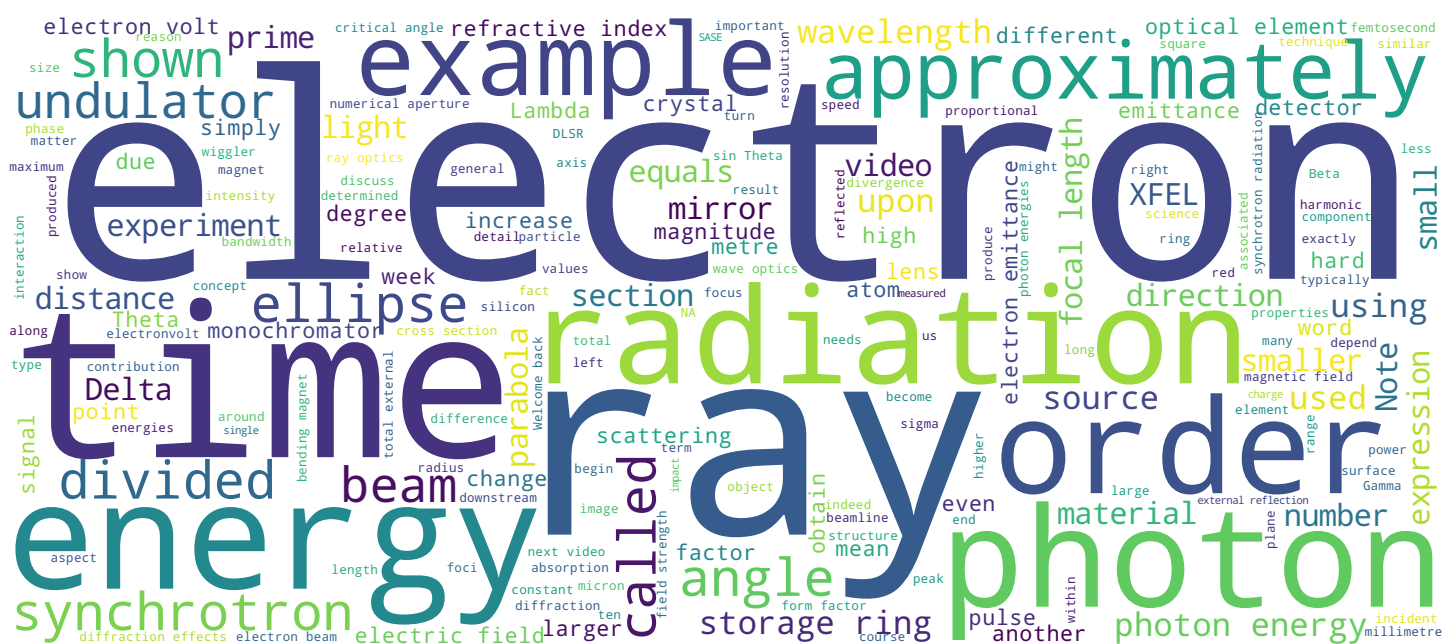


Prof. Philip Willmott



## Search MOOC



## Video



# Contents and objectives of this video



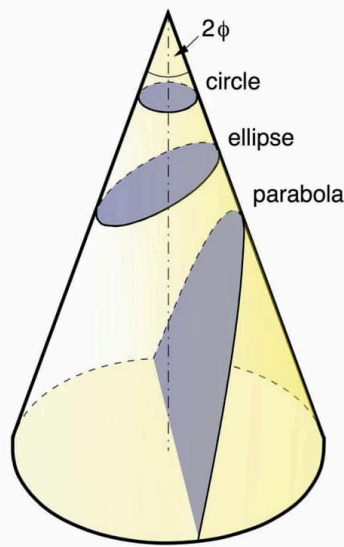
- Basic elements of optics theory I
  - Ellipses and parabolas
  - Spherical and cylindrical approximations
  - Magnification
  - The Coddington equations

In this video we will use ray or geometrical optics to describe focusing by ellipses and parabolas, and, by inference, their three-dimensional equivalent, ellipsoids and paraboloids. We introduce the concept of the numerical aperture, or NA, and show that, for small NAs, the performance of spherical or cylindrical elements is sufficient to provide good focusing. We finish with the expressions for bending radii of reflecting optical elements, via the so called Coddington equations.

Notes

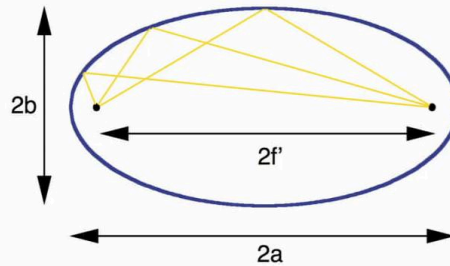
Summary



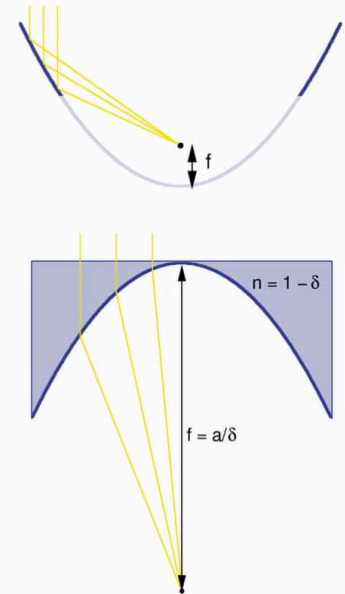


$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$f' = (a^2 - b^2)^{1/2}$$



$$y = x^2/a \quad f = \frac{a}{4}$$

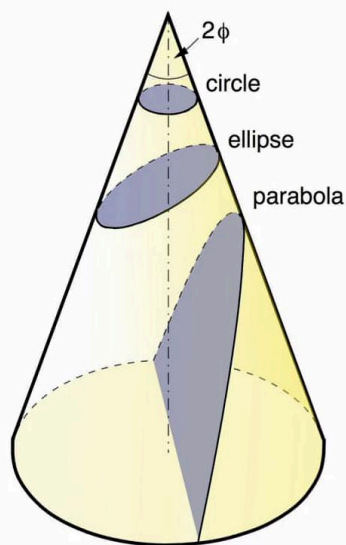


We now turn to aspects of x-ray focusing. Before we discuss specific examples, some fundamental features need to be clarified. The two shapes of primary concern for us will be the ellipse and the parabola, and, in a moment, their spherical or cylindrical approximations. Ellipses and parabolas are conic sections. An ellipse is a section for which the sectional plane subtends an angle greater than  $\Phi$  with the cone axis, where  $\Phi$  is the opening half-angle of the cone. A parabola is described by the intersection between a cone and a plane, for which the angle between the cone axis and the plane is exactly  $\Phi$ . Ellipses and parabolas are interesting because, when using ray optics, and ignoring the diffraction effects of wave optics, which is discussed later in this section, these [inaudible 00:01:42] can focus to an exact point. Their mathematical equations and expressions for the foci are provided here. Note that ray optics do not represent reality, as diffraction effects, i.e. wave optics, means that no focal spot can be of zero size. We will look at wave optics and the impact of diffraction in the next video. Nonetheless, diffraction effects excluded, reflective focusing with ellipses and parabolas is, in principle, aberration free.

Notes

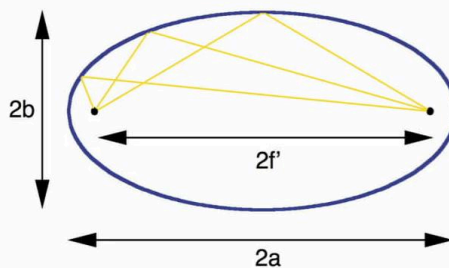
Summary



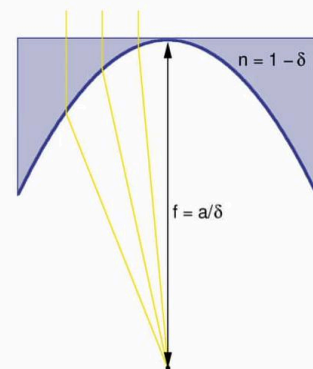
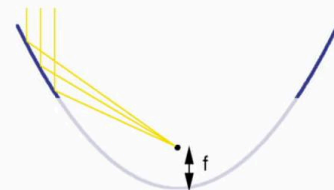


$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$f' = (a^2 - b^2)^{1/2}$$



$$y = x^2/a \quad f = \frac{a}{4}$$



Refractive focusing, using plano-parabolic lenses, shown bottom right, is, while not being perfectly aberration free, nonetheless, very precise, as we shall shortly see.

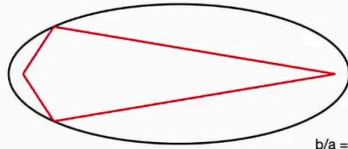
Notes

Summary

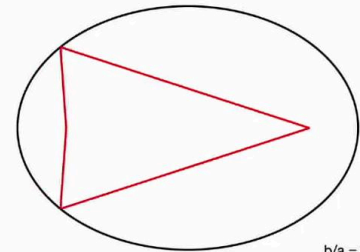




b/a = 0.100



b/a = 0.400



b/a = 0.700

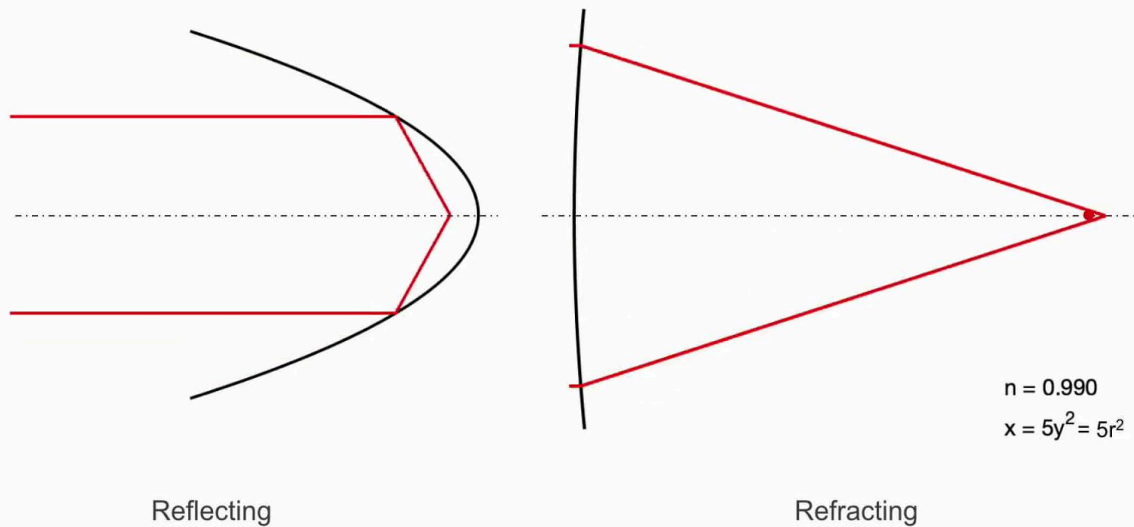
$$f' = (a^2 - b^2)^{1/2}$$

An ellipse will focus a point source to a point focus if the source lies on one of the foci of the ellipse, plus or minus  $f'$  prime. The prime is used here, as this is not the same as the focal length, denoted simply by  $f$ . The distance defined by the reflected rays between the source and image are all equal in length. Indeed, you might remember from school that one can draw an ellipse by pinning down a piece of string at two points, which then define the foci, and then putting the string under tension with a pencil, and moving this from one end to the other. Consider the symmetrical reflective path between the foci, which has its reflection in the central vertical plane. This path subtends the shallowest angle with the ellipse where it is reflected. This angle is equal to the arc tangent of  $b$  divided by  $f'$  prime, and needs to be shallower than the critical angle for total external reflection. It is therefore clear that  $b$  must be much smaller than  $f'$  prime, by about three orders of magnitude for the case of hard x-rays. In this case,  $f'$  prime is approximately equal to  $a$ , the semi-major axis. In other words, the foci are exceedingly close to the left and right-hand apexes of the ellipse, and the ellipse is much, much broader than it is high. Ellipses are therefore excellent shapes for refocusing a small source via reflection.

Notes

Summary



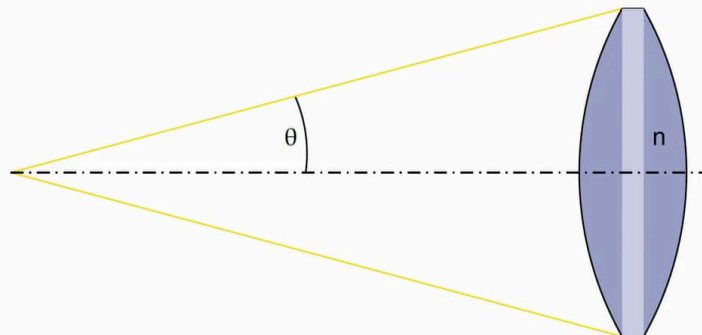


Parabolas, on the other hand, focus parallel light reflected off their inner surface to a point, as shown here. The focal length of a refractive plano-parabolic lens depends on the refractive index decrement, as well as the geometrical parameter,  $a$ . It is approximately given by  $f$  is equal to  $a$  divided by  $\Delta$ , assuming that  $r$  divided by  $f$  is much smaller than one, where  $r$  is the distance of the incident ray from the central axis. In the example shown here,  $a$  is equal to one-fifth, and  $\Delta$  is equal to 0.001. Unrealistically large for hard x-rays, but chosen here to show the deviation of the focal point with  $r$ .

Notes

Summary





$$NA = n \sin \theta$$

The numerical aperture, or NA, of an optical system, such as a lens, is a dimensionless number that characterises the range of angles over which the system can accept or emit light. In other words, the light-gathering power of an optical element. It's given by NA is equal to  $n \sin \theta$ , where  $n$  is the refractive index, which, for x-rays, can be approximated to be unity, and  $\theta$  is the half-angle of the maximum cone of acceptance of radiation the optical element can capture. If, for example, the optical element is a converging lens, then the maximum value of  $\theta$  is determined by the lens's radius,  $a$ , and the focal length,  $f$ . Assuming, as is always the case when dealing with x-rays, that  $f$  is much, much larger than  $a$ , then  $\tan \theta$  is equal to  $a$  upon  $f$ , and  $\sin \theta$  is approximately equal to  $\theta$ , which is the numerical aperture.

Notes

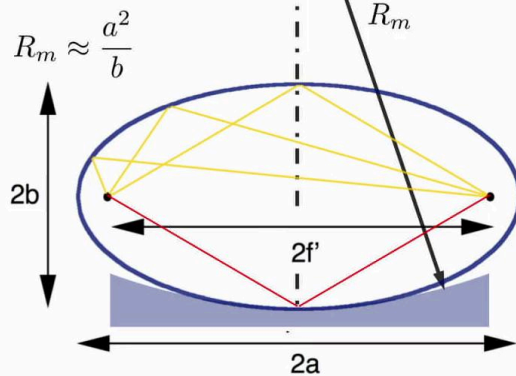
Summary





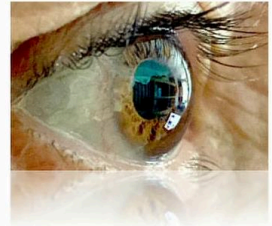
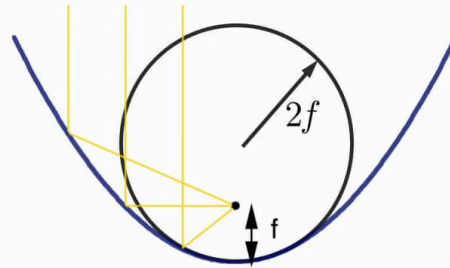
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$f' = (a^2 - b^2)^{1/2}$$



$$y = x^2/a$$

$$f = \frac{a}{4}$$



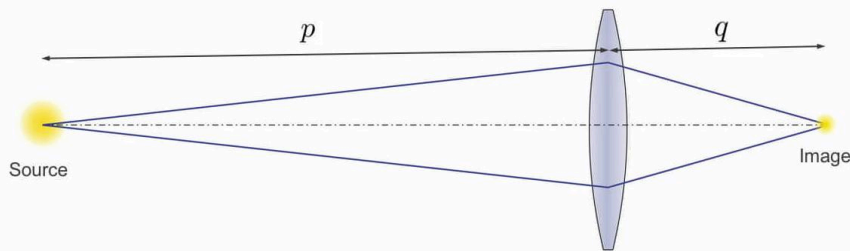
Although some optics are indeed manufactured to have either an elliptical or parabolic profile, especially secondary optical elements discussed next week, it's technologically very demanding to fabricate mirrors and larger focusing elements with exactly these shapes. Many times, one often approximates the surface of ellipses and parabolas with that of a circle or a sphere for two-dimensionally focusing elements. It can be shown, for example, that a cylindrical surface with a radius,  $R_m$ , equal to  $a^2/b$ , has only a very marginal deviation from the elliptical surface at the symmetrical reflection point shown here. Similarly, a circle with radius  $2f$  closely approximates the profile of a reflecting parabola of the form  $y = x^2/a$  at its apex. In both these cases, the numerical aperture for acceptable performance and minimal spherical aberrations is smaller than for the elliptical or parabolic elements. This is also the reason why we see better in bright sunlight than in dusky conditions. Our irises are contracted, the eye's numerical aperture is small, and the spherical aberrations are minimised.

Notes

Summary







$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$$

$$M = q/p$$

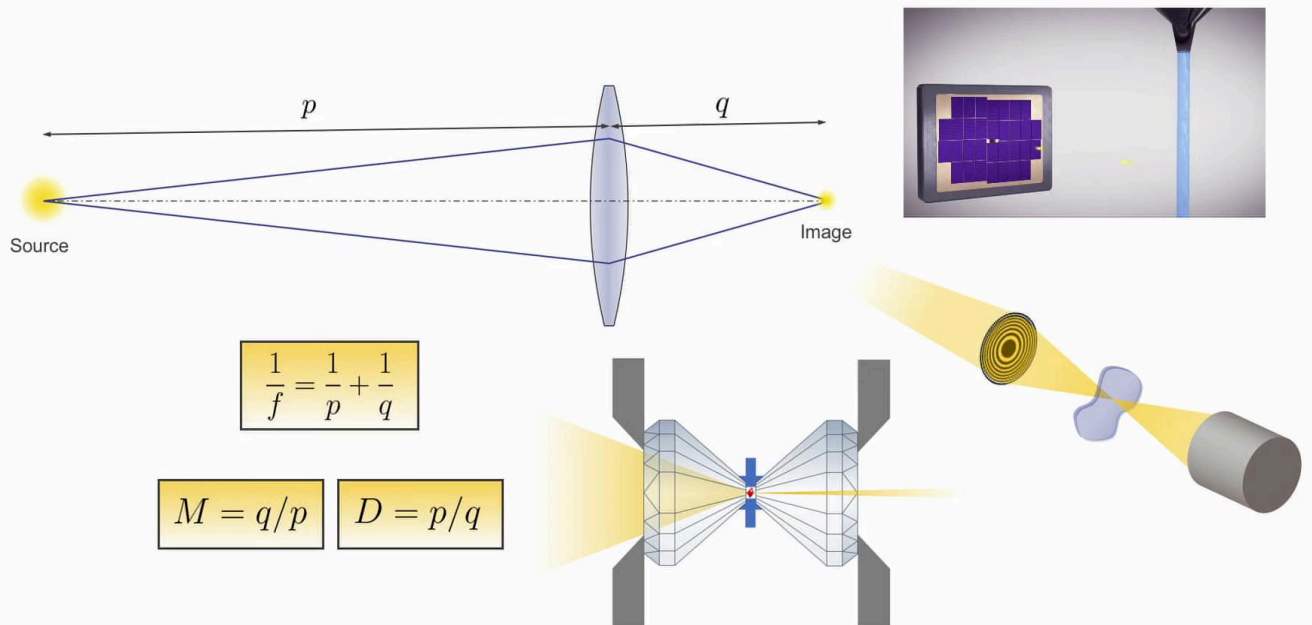
$$D = p/q$$

The lens-maker equation relates the source to lens distance,  $p$ , the lens to image distance,  $q$ , and the lens's focal length,  $f$ , using the expression one upon  $f$  equals one upon  $p$ , plus one upon  $q$ . If the source is infinitely far away, i.e. the light is parallel, then the image of the source will be at a distance,  $f$ , downstream of the lens. That is, the definition of the focal length, the distance from the lens to bring parallel rays to a focus. The magnification factor of an x-ray source is equal to  $q$  upon  $p$ , which can also be expressed as a demagnification factor,  $d$ , of  $p$  upon  $q$ . Note that the lens sketched here generically representing optics would, in fact, be a concave structure if used to converge x-rays by refraction.

Notes

Summary



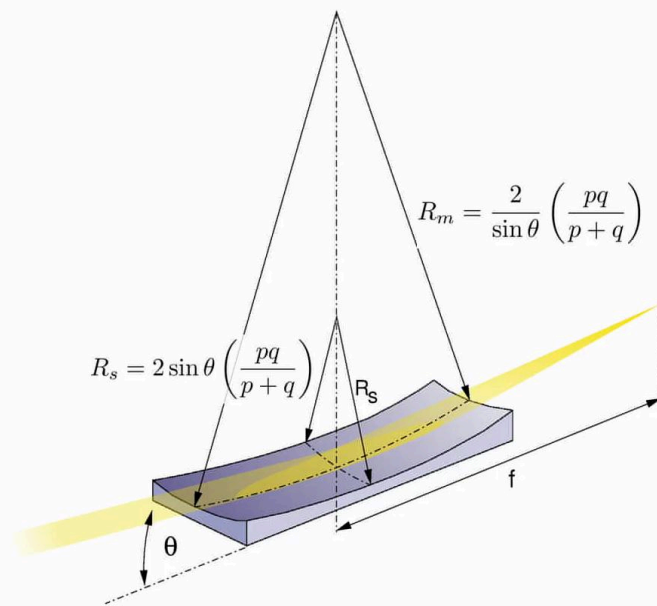


Demagnification is often used at beam lines that investigate small samples, such as in serial synchrotron crystallography, domains, as in scanning transmission x-ray microscopy, or in small environmental setups, such as x-ray diffraction of samples under pressure, in so-called diamond anvil cells.

Notes

Summary





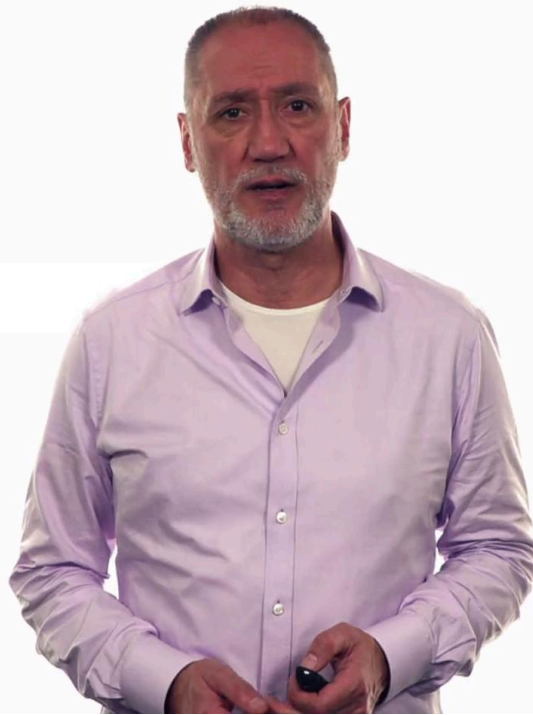
- Mirrors
  - $\theta$  small  $\sim 0.1^\circ$
  - $R_m \sim \text{km}$ ;  $R_s \sim \text{cm}$
- Diffraction gratings/crystals
  - $\theta$  large  $\sim 5 - 50^\circ$
  - $R_m \sim 10 - 100 \text{ m}$ ;  $R_s \sim 1 - 10 \text{ m}$

Lastly, we deal with the Coddington equations for the bending radii of curved mirrors. A mirror can be bent in two ways, either with the radius in a plane perpendicular to the beam direction, for which the bending radius is called the sagittal radius, or whereby the radius is in the same plane as a reflected beam, in which case it's called the meridional radius. The equations for  $R_s$  and  $R_m$  look superficially very similar. But in the case of the sagittal radius, the term  $\sin \theta$  is a numerator, whereas for the meridional, it is a denominator term. In most cases,  $\theta$  is very shallow, as it must be smaller than the angle for total external reflection. As a consequence, meridional radii are typically measured in kilometres, while the sagittal radii are a few centimetres. Any one mirror might be flat, bent only in the meridional or sagittal plane, or indeed both. We discuss these in more detail in the next section.

Notes

Summary





Until now, we've been using geometrical or ray optics, and the assumption that one can focus to a point. This description ignores the wave nature of light, and its interaction, diffraction or scattering with the optical elements it passes through or is reflected off. There is therefore a so-called diffraction limit, which arises because of this interaction, which results, for a given wavelength, in a lower limit to the resolution with which any object can be imaged. In the next video, we will discuss wave or physical optics, which includes interference and diffraction.

Notes

Summary



10m 18s